1. Show that the equation of state of an ideal gas is \( P = nkT \) even if particles in the gas are highly relativistic.

2. a) Calculate the mean molecular weights per particle, per electron and per ion for three chemical compositions: i) pure hydrogen ii) solar chemical composition, \( X = 0.7, \ Y = 0.28, \ Z = 0.02 \) and iii) for primeval chemical composition, \( X = 0.76, \ Y = 0.24, \ Z = 0 \). Show that the three molecular weights always satisfy the relation \( 1/\mu_e + 1/\mu_i = 1/\mu \).

   b) Estimate the electron pressure, ion pressure and radiation pressure at the center of the Sun. Use a temperature \( T_c = 1.6 \times 10^7 \), density \( \rho_c = 1.5 \times 10^5 \text{ kg m}^{-3} \) and composition (ii).

   c) Calculate the pressure of a non-relativistic, degenerate electron gas with density \( \rho = 10^{10} \text{ kg m}^{-3} \). Derive an upper limit for the temperature of the gas.

3. a) A low mass red giant star has a degenerate helium core in which all the hydrogen has been burnt. The degenerate electron pressure, \( p_e \), obeys the relation

\[
p_e = 2.33 \times 10^{-38} n_e^{5/3} \text{ Nm}^{-2}. \tag{1}
\]

Helium starts to burn in the core at a temperature of \( 10^8 \) K. Show that, if the density in the core is greater than \( 8.8 \times 10^7 \text{ kg m}^{-3} \), burning will commence in degenerate conditions, and that the core will undergo a helium flash. What temperature will be achieved in the flash before degeneracy is removed in a region which remains at a constant density of \( 5 \times 10^8 \text{ kg m}^{-3} \)?

b) The star has an initial mass small enough that (after all the helium has been burnt in the flash) it cannot reach the temperatures needed for carbon-burning and will turn into a white dwarf. Using the equation of hydrostatic equilibrium, derive an approximate expression for the pressure at the center of the star (use a constant density model). Derive a mass-radius relation for the white dwarf which is supported by electron degeneracy pressure \( (P_e = k \rho^{5/3}) \). What does the mass-radius relation for a white dwarf imply?