

Energy input from quasars regulates the growth and activity of black holes and their host galaxies

Supplementary Information: Methods

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The colliding galaxies in our merger simulations consist of a dark matter halo, a rotationally supported disk of gas and stars, and (in some cases) a central bulge. The structure of the galaxy models is motivated by cosmological simulations of standard Λ CDM universes where the dominant mass component is in the form of collisionless dark matter, resulting in halo density profile with a central cusp¹. The initial galaxy models are constructed using techniques in^{2,3,4}, and are initially in close to perfect dynamical equilibrium.

The dark matter and stellar components are modelled as collisionless fluids, governed by the collisionless Boltzmann equation coupled to self-gravity described by the Poisson equation. The baryonic component is followed as an ideal, monoatomic, optically thin gas, subject to radiative cooling and heating processes. The cooling processes we include are bremsstrahlung and line radiation of a primordial mix of Helium and Hydrogen, while photoheating occurs due to an imposed ionising UV background. (The latter is unimportant for the present simulations however.)

In simulations of whole galaxies in three dimensions, it is presently impossible to follow the physics of star formation and black hole accretion from first principle down to scales of individual stars or black holes. This is true both for numerical (rooted in the huge dynamic range of the problem) and physical reasons (caused by an incomplete understanding of the relevant physics). Our approach therefore resorts to the use of a subresolution model for the physics on unresolved scales^{5,4}. This is constructed based on simple physical assumptions, and intended to incorporate the effects of physics on unresolved scales onto larger, resolved scales.

Star formation and associated feedback processes are described by a multi-phase model⁵ for the star-forming inter-

stellar medium (ISM). A thermal instability is assumed to operate above a critical density threshold, producing a two phase medium consisting of cold clouds embedded in a tenuous gas at pressure equilibrium. Stars form out of the cold clouds, and short-lived stars supply an energy of 10^{51} ergs to the surrounding gas when they die as supernovae. This energy heats the diffuse phase of the ISM and evaporates cold clouds, thereby establishing a self-regulation cycle for star formation. It can be shown⁵ that this simple model reduces to an effective equation of state (EOS) for dense gas above a critical density threshold ρ_{th} for star formation. This EOS is given by

$$P = P(\rho) = (\gamma - 1)(\rho_h u_h + \rho_c u_c), \quad (1)$$

where ρ_h and ρ_c are the average densities of hot and cold phases, respectively, while u_h and u_c denote their corresponding thermal energies per unit mass. γ is the adiabatic index. The star formation rate is parameterised as

$$\frac{d\rho_\star}{dt} = (1 - \beta) \frac{\rho_c}{t_\star}, \quad (2)$$

where the star formation timescale $t_\star$ is set proportional to the local dynamical time, $t_\star = \hat{t}_\star(\rho/\rho_{\text{th}})^{-1/2}$, and β is the mass fraction of stars that explode as supernovae. The parameter $\hat{t}_\star$ is fixed by requiring that the model reproduces the star formation rates observed in isolated spiral galaxies^{6,7}, while ρ_{th} is determined self-consistently in the model by requiring that the EOS is continuous at the onset of star formation. The cloud evaporation process and the cooling function of the gas then determine the temperatures and the mass fractions in the two phases, such that the EOS of the model can be directly computed as a function of density⁵.

We model supermassive black holes at the centres of galaxies as collisionless sink particles that can accrete gas from their surroundings. Similar as for star formation, it is presently not possible to resolve the details of the accretion physics around the black hole directly in our simulations of

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colliding galaxies. Instead, we are content with a rough estimate of the accretion rate based on large-scale properties of the gas around the hole, averaged over scales of order ~ 100 pc. This assumes that the growth of the hole is ultimately limited by large-scale feeding of the hole. Note that this is physics which is well resolved in our simulations.

We relate the accretion onto the black hole (BH) to the large-scale gas distribution using a Bondi-Hoyle-Lyttleton parameterisation^{8,9,10}. In this description, the accretion rate onto the black is given by

$$\dot{M}_B = \frac{4\pi \alpha G^2 M_{BH}^2 \rho}{(c_s^2 + v^2)^{3/2}}, \quad (3)$$

where ρ and c_s are the density and sound speed of the gas, respectively, α is a dimensionless parameter, and v is the velocity of the black hole relative to the gas. We also assume that the accretion is limited to the Eddington rate

$$\dot{M}_{\text{Edd}} \equiv \frac{4\pi G M_{BH} m_p}{\epsilon_r \sigma_T c}, \quad (4)$$

where m_p is the proton mass, σ_T is the Thomson cross-section, and ϵ_r is the radiative efficiency. The latter gives the radiated luminosity, L_r , in terms of the accreted rest mass energy, i.e. $L_r = \epsilon_r \dot{M}_{BH} c^2$. We adopt a fixed value of $\epsilon_r = 0.1$, which is the mean value for a radiatively efficient Shakura & Sunyaev¹¹ accretion disk onto a Schwarzschild black hole.

We further assume that a small fraction ϵ_f of the energy released by the black hole couples to the surrounding gas. For simplicity, we assume thermal and isotropic coupling, i.e. the accreting black hole heats the surrounding gas at a rate $\dot{E}_{\text{feed}} = \epsilon_f \epsilon_r \dot{M}_{BH} c^2$. We characteristically adopt $\epsilon_f \sim 0.05$. As we show in our letter to Nature, this value results in a normalisation of the $\dot{M}_{BH} - \sigma$ relation consistent with current observations. The coefficient α in the Bondi rate is set such that a seed black hole of mass 10^5 solar masses can reach the regime of Eddington-limited exponential growth in ~ 0.5 Gyr, provided BH-feedback is unimportant.

In the final stages of our merger simulations, the cores of the galaxies coalesce to form a single stellar system. It is plausible that this also leads to the formation of a central binary system of two supermassive black holes, but it is unclear how quickly the black hole binary may be hardened by stellar-dynamical¹² or hydrodynamical processes¹³, such that the black holes eventually merge. Again, it is evident that we lack the dynamic range to study the hardening process of the black hole binary directly. We therefore assume that binaries of supermassive black holes merge efficiently. In practice, we allow two black hole particles to merge once their separation has fallen to our spatial resolution limit, and their relative speed lies below the local soundspeed of the gas.

Numerically, we use the N-body method to solve the collisionless dynamics of stars and dark matter. In it, the

mass distribution is discretised in terms of particles, which can be viewed as providing a Monte-Carlo sampling of 6-dimensional phase space. Following the equations of motion of these particles gives then an approximate solution to the Boltzmann equation, provided gravity is softened on a small scale to prevent collisional behaviour. This softening is of order 0.1 kpc in our simulations (depending slightly on particle number) and provides a lower spatial resolution limit. For the treatment of hydrodynamics, we also use a particle-based approach, in the form of smoothed particle hydrodynamics¹⁴. This Galilean-invariant, Lagrangian method has found widespread use in astrophysics thanks to its ability to easily adjust to a large dynamic range in spatial scales.

Our simulation code is called GADGET2¹⁵, which is well tested and presently belongs to the most widely employed codes in numerical cosmology. For the computation of self-gravity, the code uses a hierarchical multipole expansion of the gravitational field based on a tree-algorithm¹⁶. This method provides accurate forces down to the gravitational softening scale, free of geometric restrictions and anisotropic force errors. Mesh-based gravity solvers could be used in principle instead of the tree-algorithm, but it is hard to make them work efficiently on the large dynamic range encountered in the galaxy collisions we follow.

Our numerical implementation of SPH uses a formulation that manifestly conserves energy and entropy despite the use of fully adaptive SPH smoothing lengths¹⁷. Radiative cooling and heating processes are solved on a per-particle basis assuming collisional ionisation equilibrium¹⁸. Star formation is modelled with the multi-phase model described above, where highly overdense gas is pressurised by an effective equation of state. Independent collisionless star particles are spawned stochastically out of the gas⁵, with a rate that follows (on average) the estimated local star formation rate. Similarly, accretion onto supermassive massive black holes is treated in terms of sink particles, using the model described above. These black hole particles estimate the local gas density by SPH kernel estimation in the same way as it is done for ordinary gas particles. Gas particles from the smoothing region are absorbed stochastically by the black in accordance with its estimated accretion rate. The feedback energy of the black hole is injected into the gas in the local smoothing region as thermal energy in a kernel-weighted fashion. Further details on the numerical method are given in⁴.

We note that our physical model does not rely on a particle-based numerical treatment of the dynamics. In principle it is possible to study the model with other numerical schemes, including Eulerian hydrodynamic solver. However, mesh-based approaches are substantially more difficult to adapt to the dynamic range required by 3D models of whole galaxies, and in particular, to the additional challenges posed by the high-speed motion of these systems through space.

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