The endpoints of stellar evolution

Uncertainties in the boundaries due to eg,

- Treatment of convection.
- At high masses, extent of stellar winds.

$M < 0.08M_\odot$	Brown dwarf
$0.08M_\odot < M < 0.5M_\odot$	Central hydrogen burning
	Formation of a degenerate core
	No helium ignition
	Helium white dwarf
$0.5M_\odot < M < 2M_\odot$	Central hydrogen burning
	Helium flash
	CO white dwarf
$2M_\odot < M < 8M_\odot$	Central hydrogen burning
	Helium ignites in nondegenerate core
	CO white dwarf
$8M_\odot < M < 20M_\odot$	Numerous burning phases
	Bulk of heavy element enrichment
	from $M > 10M_\odot$
	Type II supernova
	Neutron star
$M > 20M_\odot$	**Black hole**

Answer depends on the star's mass

Star exhausts its nuclear fuel -
can no longer provide pressure to support itself.
Gravity takes over, it collapses

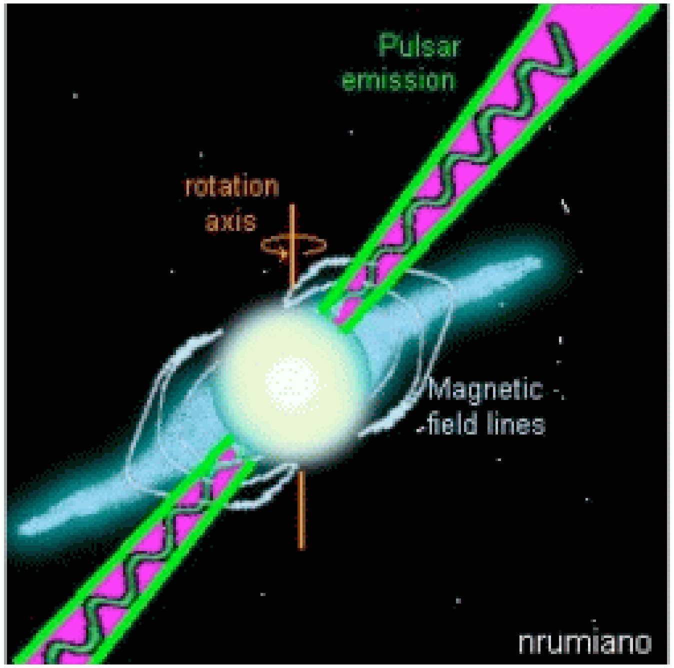
Low mass stars (< 8 Msun or so): End up with 'white dwarf' supported by degeneracy pressure.

Wide range of structure and composition!!!

Answer depends on the star's mass

Star exhausts its nuclear fuel -
can no longer provide pressure to support itself.
Gravity takes over, it collapses

Higher mass stars (8-25 Msun or so):
get a 'neutron star' supported
by neutron degeneracy pressure.

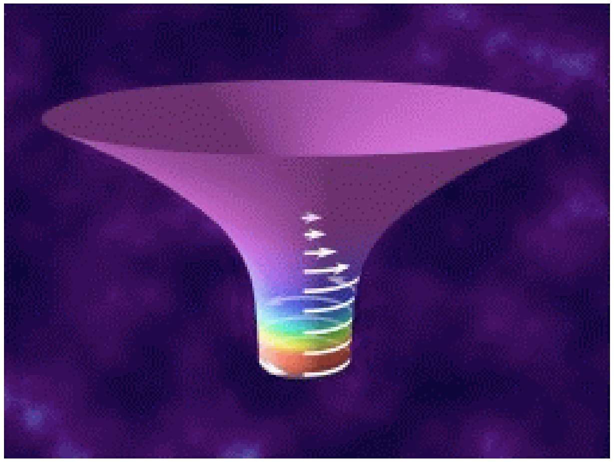


Wide variety of observed properties

Answer depends on the star's mass

Star exhausts its nuclear fuel -
can no longer provide pressure to support itself.
Gravity takes over, it collapses

Above about 25 Msun:
we are left with a 'black hole'



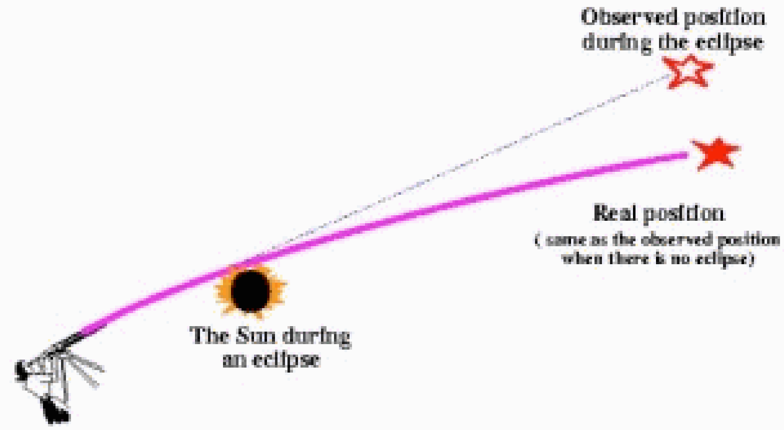
Black holes are astounding objects because of their simplicity (consider all the complicated physics to set all the macroscopic properties of a star that we have been through) - not unlike macroscopic elementary particles!!

According to general relativity, the black hole properties are set by **two numbers**: *mass* and *spin*

Heuristic idea: "object" with gravity so strong that light cannot escape

Key concepts from general relativity that drive this idea:

1. Bending of light by gravity

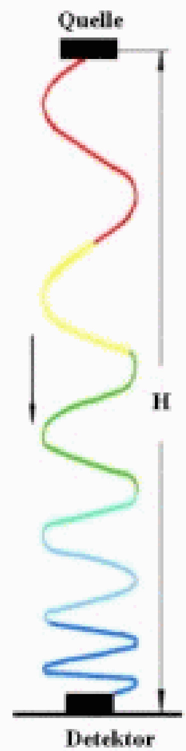


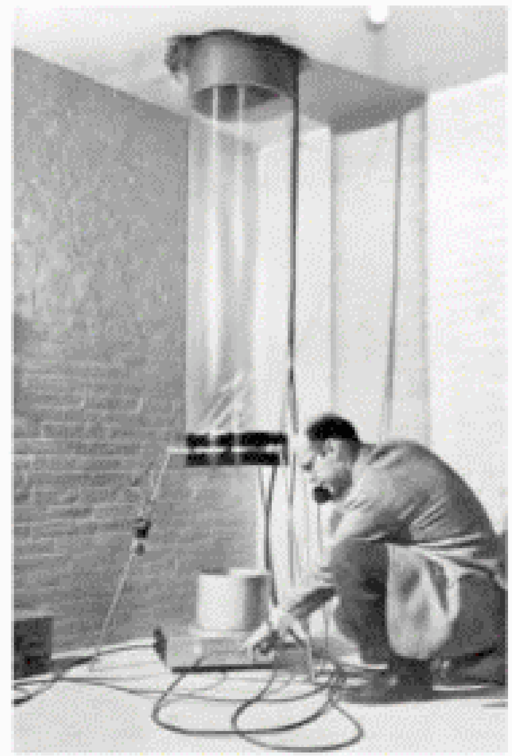
Heuristic idea: “object” with gravity so strong that light cannot escape

Key concepts from general relativity that drive this idea:

1. Bending of light by gravity

2. Gravitational redshift



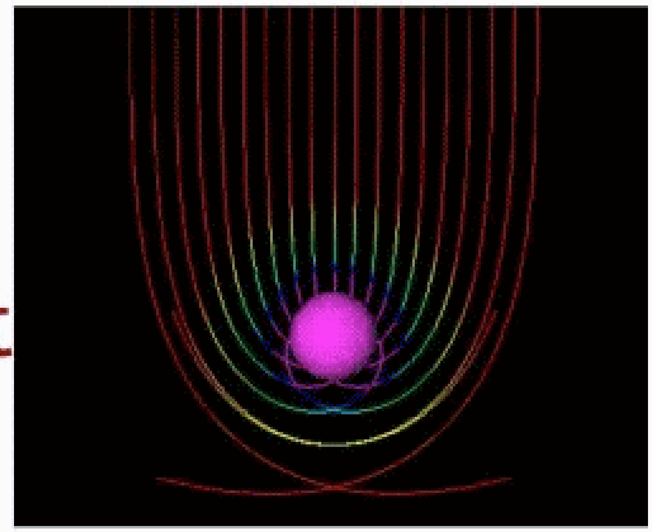


change of photon energies towards the gravitating mass
the opposite when emitted

Pound-Rebka experiment
used atomic transition of Fe

Black holes: Concepts run amok!

Light bending so fierce that “bent” trajectories close …



Gravitational redshift powerful enough to *completely* “drain” photon energy!

Relativity units: G=c=1

Important physical content for BH studies:
general relativity has no intrisic scale

instead, all important lengthscales and timescales are set by - and become proportional to the mass.

Basic properties in GR

Simplest example: Schwarzschild solution, represents a non-rotating BH:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Some odd features immediately apparent:

- $r = 2M$ – apparent divergences.
  - Coordinate singularity, corresponding to "event horizon": Light cannot escape from this radius. Metric pathology can be eliminated by changing coordinates (though physical meaning is not).
- $r = 0$ – **more** divergences.
  - **Physical** singularity: *cannot* be removed.

Trick…

Newtonian escape velocity

Calculation by Reverend John Michell

[Philosophical Transactions of the Royal Society of London **74**, 35-17 (1784)]

"Popularized" by Laplace

[*Exposition du Systeme du Monde*, 1796]

Trick: Abuse escape velocity formula.

$$\frac{1}{2}v_{esc}^{2}=\frac{GM}{R}$$

At what value of *R* does $v_{esc} = c$?

Newtonian escape velocity

Calculation by Reverend John Michell

[Philosophical Transactions of the Royal Society of London **74**, 35-17 (1784)]

“Popularized” by Laplace

[*Exposition du Systeme du Monde*, 1796]

Trick: Abuse escape velocity formula.

$$R = R_{crit} = \frac{2GM}{c^2}$$

Schwarzschild metric: Details

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Easy to verify by direct substitution this is an exact solution of the vacuum (T_{ab} = 0) Einstein equations.

BUT: Is it a ***meaningful*** solution?

In other words, are the features of this mathematical solution features that are likely to be shared by objects in the real world?

Schwarzschild metric in Schwarzschild coordinates:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

- Spherically symmetric.
- Coordinate r labels surfaces of constant area, $A = 4\pi r^2$.
- Coordinate r does ***not*** label distances in a simple way! Must integrate.
- Coordinate t corresponds to time as measured by distant ($r >> M$) observers.

The event horizon

Big question: What is up with that singular-ish behavior at $r = 2M$?

Best way to answer: Examine what happens to "stuff" as it moves around near this region.

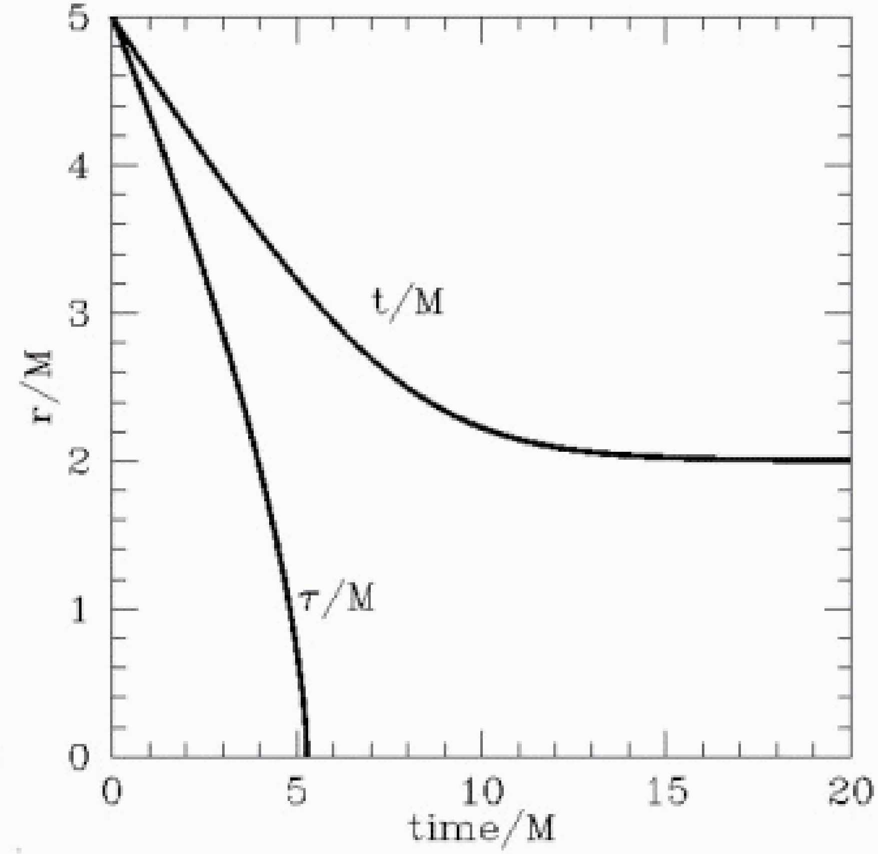
Specifically: Drop a particle from some finite starting radius r_0 (no angular momentum - purely radial trajectory).

Integrate the geodesic equation, calculate its position r as a function of "distant observer time t" and as a function its ***own*** time τ (also called "proper time".

The event horizon

A body that falls from finite radius passes through r = $2M$ in finite *proper* time, eventually reaching r = 0 …

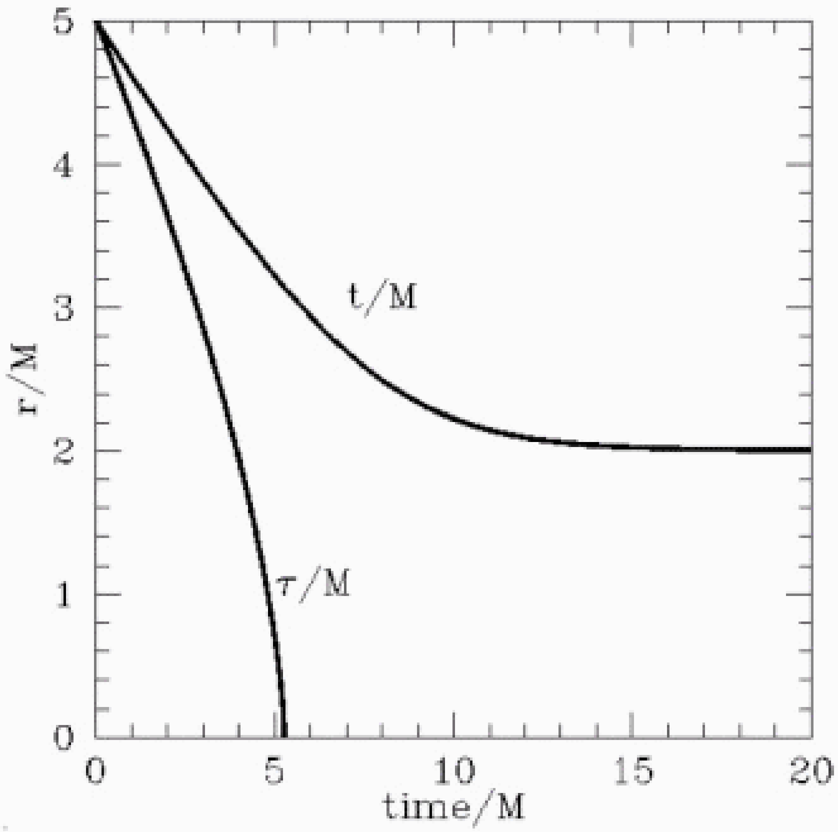
But, it takes ***infinite*** coordinate time to reach r = $2M$!!



The event horizon

This means that distant observers *never see the infalling body cross $r = 2M$.*

The infalling body itself passes through there with no problems - doesn't notice anything 'special'.



Examine the energy…

The event horizon

Result: For a photon emitted at some radius *r* and observed very far away,

$$\frac{E_{\rm obs}}{E_{\rm em}} = \sqrt{1 - \frac{2M}{r}}$$

Radiation emitted near $r = 2M$ is highly redshifted, with $r = 2M$ corresponding to a surface of ***infinite redshift*.**

Coordinate pathologies are due to this infinite redshifting as we approach $r = 2M$.

Summary

The Schwarzschild metric is a perfectly viable candidate description for highly condensed objects in the universe.

- Exterior spacetime describes *any* spherically symmetric gravitating body.
- Event horizon at $r = 2M$ is odd, but reasonable: From exterior perspective, just the extreme limit of gravitational redshift.
- Reason that it's a horizon: Anything that passes inside $r = 2M$ cannot *ever* get out.

Schwarzschild from "normal" stuff?

Schwarzschild is mathematically viable ... but can we actually get it from reasonable initial conditions? Put it another way:

Does smooth, normal "stuff" actually collapse into this odd solution??

Yes.

Reference: J. R. Oppenheimer and H. Snyder, "On Continued Gravitational Contraction", Phys. Rev. **56**, 455 (1939).

Why do we believe that black holes are inevitably created by gravitational collapse?

Rather astounding that we go from such complicated initial conditions to such "clean", simple objects!

Oppenheimer-Snyder Collapse

Idealized but illustrative, totally *analytic* calculation of gravitational collapse.

Examine the collapse of a pressureless "dustball". Initial data is totally smooth. Ball's radius shrinks (increasing in density) due to self gravity.

Surface passes through $r = 2M$ without incident: A black hole is left behind.

Oppenheimer-Snyder Collapse

Some details of the calculation:

Use Friedman-Robertson-Walker metric to describe the interior ("collapsing universe cosmology")

Use Schwarzschild metric to describe the exterior (Birkhoff's theorem)

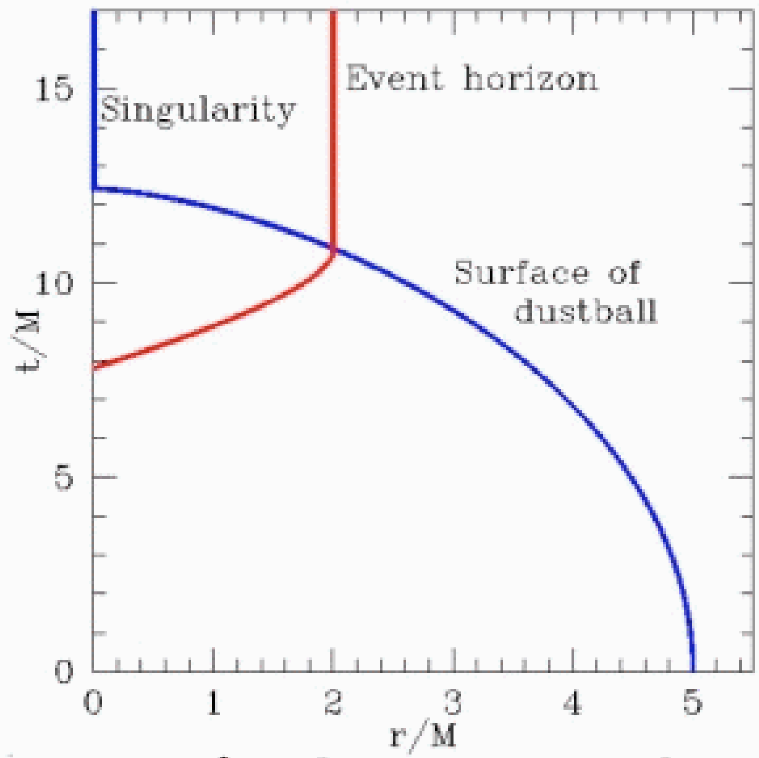
Match: Require that the metric components in the interior and the exterior forms agree at the surface.

O-S Collapse: More details

Spherical collapse with pressure can't be done analytically ... but, can be done numerically without too much problem.

Always find that, if pressure is not sufficient, the object collapses through $r = 2M$ and leaves a black hole behind.

Oppenheimer-Snyder collapse



Black holes are a generic feature of spherical gravitational collapse!!