

Stellar parameters: observational constraints

How do we describe a star? The internal properties of stars can be primarily described by a few parameters, **mass M** , **radius R** , **temperature T** , composition and the related total **(bolometric) luminosity L** (total energy output integrated over all wavelengths, units erg s^{-1})

There are also several important external parameters, such as distances and their motion in space. The distance is essential for determining the internal properties of stars.

DISTANCES:

The distance to stars can be measured with a number of different methods.

The most fundamental one is the parallax is also the simplest and purely geometrical method.

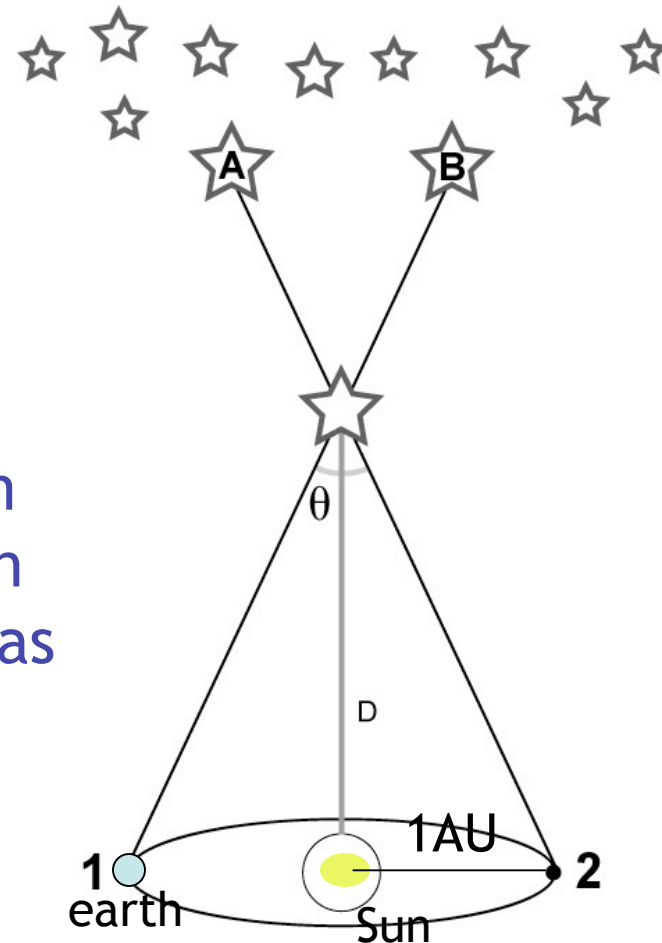
The viewing angle of the stars from the earth change due to the motion around the sun (use small angles - as the parallax

$$d = 1 \text{ AU} / \theta$$

$$1 \text{ AU} = 1.5 \times 10^{13} \text{ cm}$$

when the parallax is in arcsec the corresponding distance is Called parsec (pc). Since 1 radian = $180/\pi \times 60 \times 60 \sim 2 \times 10^5$ arcsec

$$\rightarrow 1 \text{ pc} = 2 \times 10^5 \text{ AU}$$



Quiz: A star at 500pc will have a parallax of arcsec ?

The recent satellite Hipparcos can measure parallax as accurate as 0.002 arcsecond, so we can roughly measure the star distances out to few hundred pc. future satellite missions such as GAIA will be able to measure parallax to even higher accuracy, hence they can measure distances to all bright stars in our galaxy.

Luminosities and related concepts

- Stars radiate and we see the photons from them. the luminosity of a star is defined as the energy radiated per unit time - second.

$$L = \text{energy/second} \quad [\text{erg s}^{-1}]$$

- The flux is the luminosity per unit area. The flux by

$$f = L / 4\pi d^2 \quad [\text{erg s}^{-1} \text{ cm}^{-2}]$$

The star emits in all directions so the area is the surface area of a sphere with radius equal to the distance, d , to the star.

For our Sun, $L_{\odot} = 3.8 \times 10^{33} \text{ erg s}^{-1}$ (bol. luminosity)

Magnitudes and colors

In astronomy, for historical reasons, luminosity and fluxes are frequently measured is so-called absolute and apparent magnitudes.

For a star at distance, d , with an observed flux, $f = L / (4\pi d^2)$ the **apparent magnitude**,

$$m = -2.5 \log f + \text{constant}$$

i.e a star is 5 magnitudes brighter (smaller magnitude) has 100 times the flux. For two star with fluxes f_1 and f_2 , the apparent magnitudes m_1 , m_2 ,

$$f_2 / f_1 = 10^{-(m_2 - m_1) / 2.5} = 2.512^{(m_1 - m_2)}$$

Central wavelenghts, λ , for common bands are:

B-band - the blue part of the electromagnetic radiation with $\lambda = 445 \text{ nm}$

V-band - the 'visual' part
with $\lambda = 551 \text{ nm}$

The zero points are different for each band - need to look these up as required

Absolute magnitude M is defined as the apparent magnitude the star would have at a fiducial distance (10 pc).

Let f_{10} be the flux when star is at 10pc, mag M

Let f_d be the flux when star is at distance d , mag m

Then,

$$f_{10}/f_d = (d / 10\text{pc})^2 = 10^{-(M-m)/2.5}$$

$$m-M = 5 \log (d/10\text{pc})$$

$m-M$ is the distance modulus.

To compare with theory, we are most interested in the **bolometric magnitude** M_{bol} - the absolute magnitude that would be measured by a bolometer sensitive to all wavelengths.

Define the bolometric correction via:

$$BC = M_{\text{bol}} - M_V = m_{\text{bol}} - m_V$$

Quiz: what is the apparent magnitude of the sun in the B-band, $M_{B(\text{sun})} = 5.48$

Mass

The mass of stars are difficult to measure. They can be determined for binary systems when the orbital motion is known. Most stars (~ 80%) of stars are in binary systems. In this case the inclination must be known e.g. in eclipsing binaries.

For the Sun: $M_{\odot} = 1.989 \times 10^{33} \text{ g}$

Other stars: $0.1 M_{\odot} < M < 60 M_{\odot}$

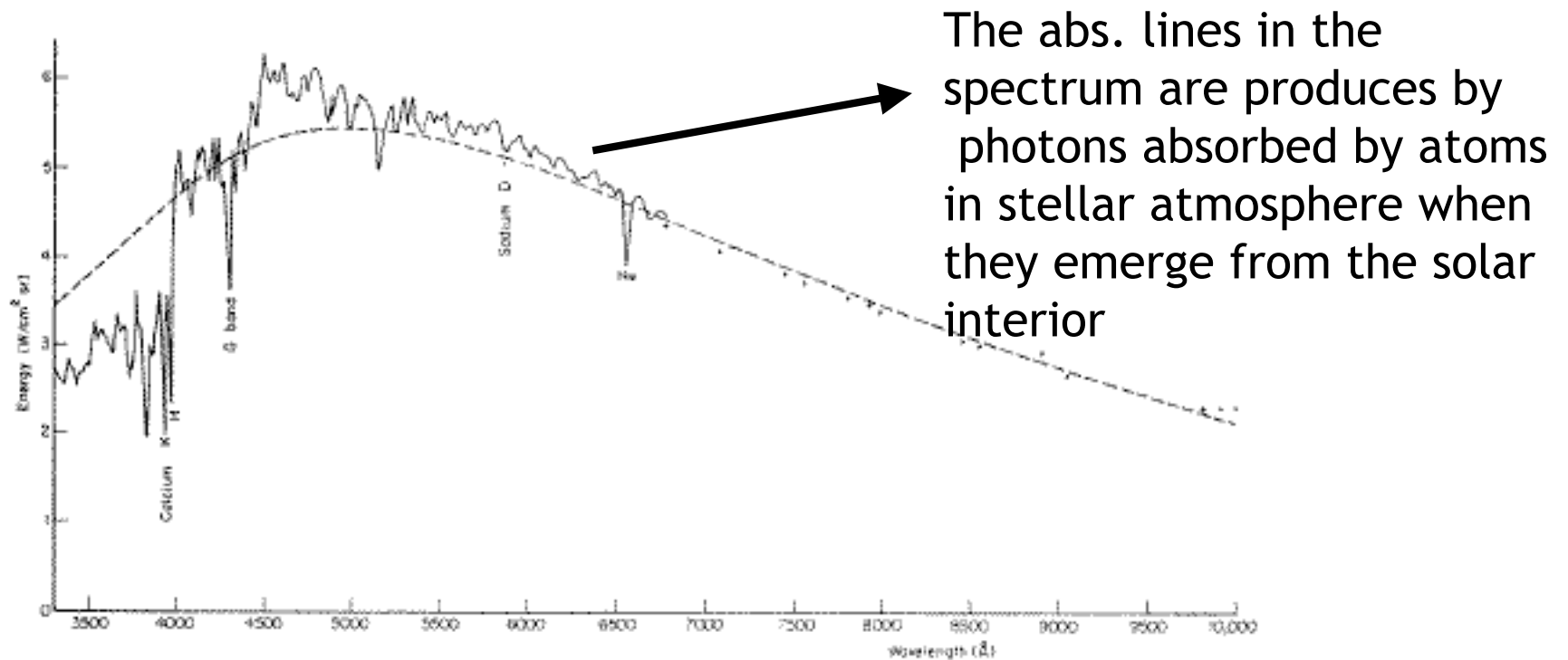
Effective temperature and composition

The energy distribution of a star as a function of wavelength or frequency is called the spectrum. The spectra of stars can be crudely approximated to a blackbody. Irrespective of the actual form of the spectrum, we define the effective temperature, T_{eff} as the temperature of a blackbody with the same luminosity/unit surface area. Stefan-Boltzmann law then gives:

$$L = 4 \pi R^2 \sigma T_{\text{eff}}^4 \quad \text{where } \sigma = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$$

Where stars lie in the L, T_{eff} plane is of greatest interest theoretically. Often L is not known (d is not known) and only information maybe color

Solar spectrum



The solar spectrum in low-resolution can be approximated by a blackbody (dashed line) of $T_{\text{eff}} = 5770 \text{ K}$

COLORS

The effective temperature of a star can also be crudely measured using the so called color index. A color index is simply defined as the magnitude difference between two colors.

$$B - V = m_B - m_V$$

Note that *colors are distance independent* and are related to the effective temperature because Stars radiate approximately as blackbodies.

Colors are zero for a Vega like star with $T_{\text{eff}} \sim 9500\text{K}$
Then $(B-V) < 0$ for hotter T and $(B-V) > 0$ for cooler T

Spectral Class

Stars can be divided into various classes according to their spectra. The sequence is historically termed:

O B A F G K M

Corresponding to effective temperatures:

40000K 20000K 10000K 6700K 5500K 4500K 3500K

Early type Blue ----->>> red (B-V) Late type

L and T dwarfs are cooler, and generally substellar.

Each class is divided into subclasses e.g. sun is G2.

The BC is zero for F5. B=V for A0

Range of stellar parameters

What are the typical values of the stellar parameters?

Parameter	Sun	Stars
Radius	$R_{\odot} = 6.96 \times 10^{10} \text{cm}$	$10^{-2} - 10^2 R_{\odot}$
Mass	$M_{\odot} = 1.99 \times 10^{33} \text{g}$	$10^{-1} - 10^2 M_{\odot}$
T_{eff}	$T_{\text{eff}\odot} = 5770 \text{ K}$	$10^3 - 10^5 \text{ K}$
Luminosity	$L_{\odot} = 3.83 \times 10^{33} \text{erg/s}$	$10^{-5} - 10^6 L_{\odot}$

- Sun is a typical star with intermediate values
- Mass has a lower and upper cutoff. This is because lower mass cannot initiate nuclear burning and higher are unstable
- The luminosity range varies 11 order of magnitude, much larger than any other parameter!

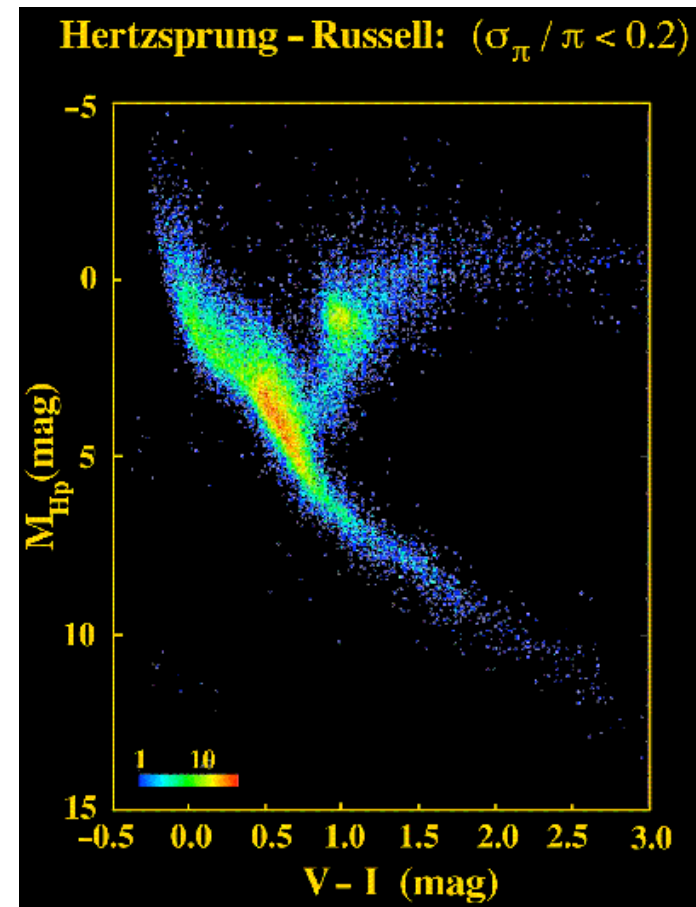
Hertzsprung-Russell (HR) diagram

Originally this was a plot of apparent magnitude versus color - Hertzsprung 1911 - for stars within a cluster.

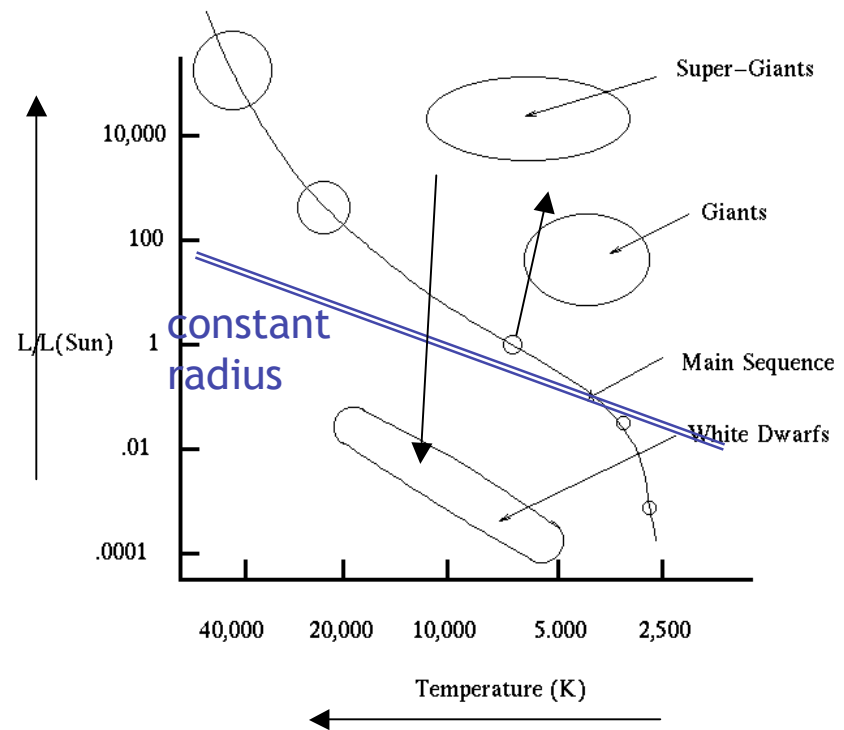
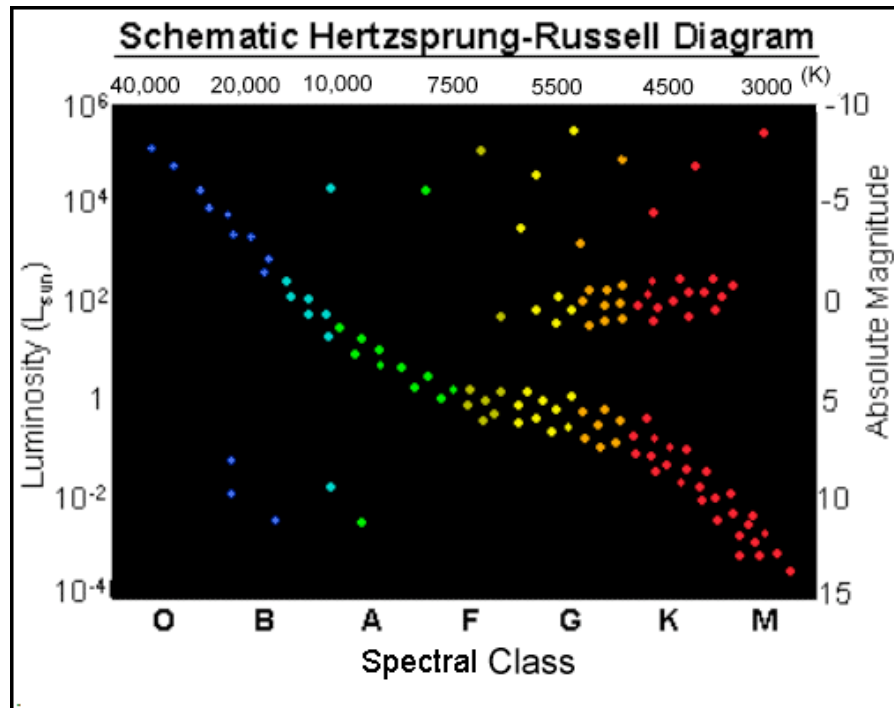
The figure shows the HR diagram for 41453 nearby stars with accurate d measured by Hipparcos satellite. The x-axis is $V-I$ a color index similar to $B-V$

Observe main sequence (top left bottom right).
right of MS, evolved stars and bottom left
stellar remnants

Nowadays any plot of L vs T_{eff}
Or M_V vs color is called HR diagram



Hertzsprung-Russel (HR) Diagram



Stars become more luminous from bottom to top
hotter and bluer from right to left
More luminous stars are more massive (see later)

HR diagram of star clusters

Stars are not distributed uniformly in space, they are highly clustered. For a cluster we can assume that the stars are approximately to the same distance, all have the same origin and hence the same age and composition. They only differ in their masses!

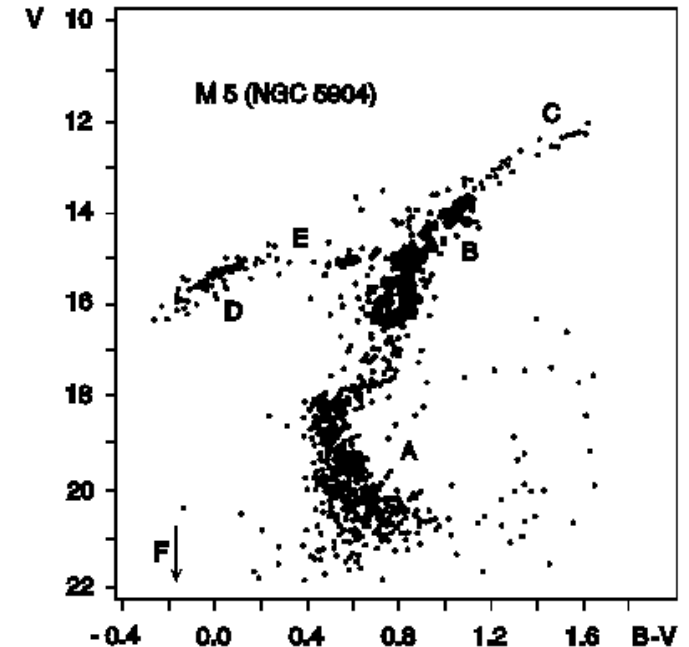
HR diagrams of clusters show less scatter than for other groups of stars because errors in d are absent

HR diagrams of clusters show a turn-off from the main sequence - no stars on the MS above a certain L .

Globular clusters:

Globular clusters are very dense star cluster with 10^5 - 10^6 stars with size of a few pc.

- A- main sequence
- B-red giant branch
- D-horizontal branch



All galactic globular
Clusters have similar
MS turn off points
(smaller ages) and a
giant branch joining the MS.

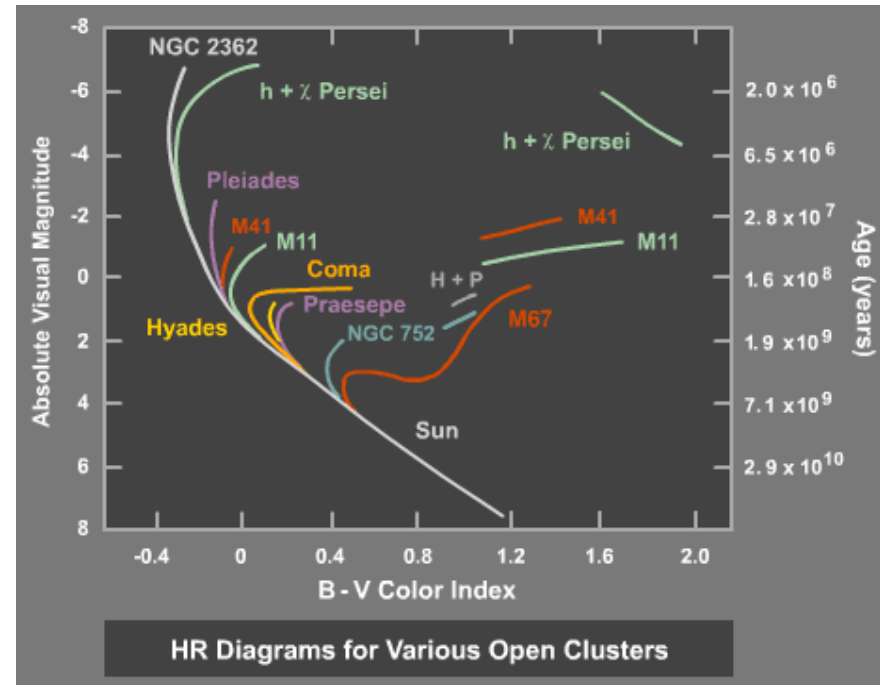
They also have a horizontal branch near the top of the giant branch
across the MS.

Stars in GCs are old: high mass stars have short lifetimes
Stars have already evolved away from MS - forming
RedGiants and the Horizontal branch.

Detailed studies of HR diagrams in GCs can tell us about the ages

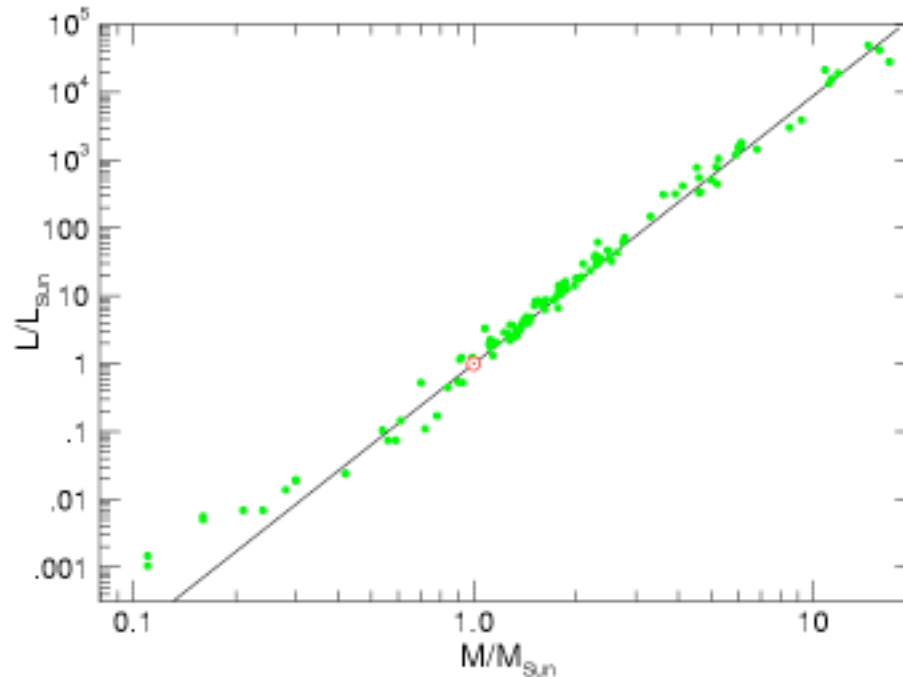
Open clusters

Have only hundreds of stars, they are a few pc across.
e.g pleiades open cluster



HR diagrams of open clusters v. different to GCs.
Why are stars clustered on the MS here? Simple:
they are young. Not had enough time to evolve of
the main sequence - fitting for the turn-off points
yields and age estimate

Mass-luminosity relation



There is an empirical relation between M and L for MS stars. Any theory of stellar evolution must explain this relation. We have:

$$\text{Log} (L/L_{\odot}) = \beta + \alpha \text{log} (M/M_{\odot})$$