33-467: Astrophysics of Stars and the Galaxy
Due: Wednesday 9th November.

Problem Set 8

1. The rates of energy generation by proton-proton and CN-cycle processes are given by

$$E_{PP} = 0.25\rho X_H^2 \left(\frac{10^6}{T}\right)^{2/3} \exp[-33.8 \left(\frac{10^6}{T}\right)^{1/3}]\ Jkg^{-1}sec^{-1} \tag{1}$$

$$E_{CNO} = 8 \times 10^{20}\rho X_H X_{CN} \left(\frac{10^6}{T}\right)^{2/3} \exp[-152.3 \left(\frac{10^6}{T}\right)^{1/3}]\ Jkg^{-1}sec^{-1} \tag{2}$$

where X_H and X_{CN} are the fractional abundances of hydrogen and of (C + N), respectively.

Find the rate of energy generation, the luminosity and the surface effective temperature and estimate how long hydrogen burning can continue at the present rate, for each of the following model stars:

(a) A B1 main sequence star, $M = 10M_\odot$, $R = 3.6R_\odot$, with energy generation in a central region of $0.15R$, of uniform density $\rho = 10^4$ kg m^{-3}, temperature 2.7×10^7K and composition $X_H = 0.88$ and $X_{CN} = 0.005X_H$.

(b) An initial solar model, with the same composition as (a) and energy generation in a central $0.2R$, at uniform density $\rho = 5.5 \times 10^4$ kg m^{-3} and temperature $T = 10^7$ K.

(c) A red giant star of $R = 100R_\odot$ where the main energy source is in a thin hydrogen burning shell outside an inert helium core. The shell is between radii 1.8×10^7 and 2.0×10^7 m, and of mean density 5×10^4 kg m^{-3}, temperature 5×10^7 K and composition $X_H = 0.5$, $X_{CN} = 10^{-3}X_H$.

Find the temperature at which the rate of energy generation due to proton-proton reactions is equal to that from CN-cycling for a gas with $X_{CN} = 10^{-3}X_H$.

2. Find the classical distance of closest approach for two protons with an energy approach equal to 2 keV.

If the probability of penetrating the Coulomb barrier for two nuclei with charge numbers of Z_A and Z_B is

$$P(E) = \exp[-b/E^{1/2}], \tag{3}$$

where,

$$b = 31.28 Z_A Z_B \mu^{1/2} keV^{1/2} \tag{4}$$

and E is the relatively energy of the particles, find the penetration probability for the same two protons.

Given that the Gamow peak is determined by the function,

$$f(E) = \exp\left[-\frac{E}{kT}\right] \exp\left[-\frac{b}{E^{1/2}}\right], \tag{5}$$

calculate the energy of the maximum of the Gamow peak for a temperature of 1.5×10^7K and the penetration probability at this energy.

[μ is the reduced mass of the two particles in atomic mass units, 1 keV = 1.602×10^{-16} J]