33-467: Astrophysics of Stars and the Galaxy
Due: Wednesday 2nd November.

Problem Set 7

1. Recall that the adiabatic index, γ is the ratio of the heat capacities at constant pressure and at constant volume. Show that, for an ideal classical gas, the critical temperature gradient for the onset of convection can be written as

$$\left[\frac{dT}{dr}\right]_{\rm conv} = -\frac{g}{C_P}, \tag{1}$$

   where C_P is the thermal capacity per unit mass at constant pressure and g is the acceleration due to gravity.

2. For a star of 1 $M_\odot$, show that the Eddington luminosity is about 10^{31} W.

   Consider a binary system in which matter accreting onto a compact object leads to a luminosity equal the Eddington luminosity (when matter is accreted the gravitational energy is converted into radiation). Show that if the compact object is a white dwarf ($R = 10^4$ km) then it radiates in the ultraviolet, but if it is a neutron star ($R = 10$ km) then it radiates in the X-ray band.

3. The physical quantities near the center of a star are given in the following table. Neglecting radiation and assuming a mean molecular weight $\mu = 0.7$, determine energy transport is convective or radiative.

r	$m(r)$	$L(r)$	$T(r)$	$\rho(r)$	κ
$0.1R_\odot$	$0.028M_\odot$	24.4 $L_\odot$	2.2×10^7 K	3.1×10^4 kg m^{-3}	0.04 m^2 kg^{-1}