33-467: Astrophysics of Stars and the Galaxy
Due: Wednesday 5th October.

Problem Set 4

1. Consider a photosphere composed of pure neutral hydrogen. At what temperature is the density of atoms in the n=2 excited state and in the n=1 ground state equal ($N_2/N_1 = 1$; recall $g_n = 2n^2$)?

   Assume our photosphere has a constant electron pressure of $P_e = 2$ Pa (2 N/m^2). Use the Saha equation to find the temperature at which the ionized and neutral fractions are equal $N_{II}/N_I = 1$.

2. Consider helium in a photosphere of temperature T. The helium atoms will be in ionization equilibrium between neutral He I, singly ionized He II, and doubly ionized He III. Note that helium has a nuclear charge of Z=+2, and the ionization potentials of He I to He II is 24.5 eV, while that from He II to He III is 54.4 eV. Calculate the fractions of the populations in the various ionization states N_{II}/N_I and N_{III}/N_{II} for T = 10000 K, 20000 K, 30000 K and 40000 K. Comment on where the Helium Balmer series strength will be a maximum.

   (Hint: assume 2 Pa for the electron pressure, and you will have to put something in for the partition functions, you can either think carefully about this or if you are inclined, you might go to the library and see if you can find out what the He I,II,III partition functions should be!)

3. Calculate the wavelength $\lambda(\nu_{max})$ at which the Planck function

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \left(exp(h\nu/kT) - 1\right)^{-1} \tag{1}$$

   takes its maximum value. Derive a formula for $B_\lambda(T)$, the Planck function per unit wavelength interval, and calculate the wavelength λ_{max} at which it takes it's maximum value. Why are $\lambda(\nu_{max})$ and λ_{max} different? Calculate the characteristic temperature for the following valies of λ_{max}. noting the region of the electromagnetic spectrum involved and typical astrophysical regions where each temperature is found: $\lambda_{max} = 0.3$mm; 30um; 3000A; 300A; 30A; 3A; 0.3A.