33-467: Astrophysics of Stars and the Galaxy
Due: Monday 26th September.

Problem Set 3

1. Show that the equation of state of an ideal gas is $P = nkT$ even if particles in the gas are highly relativistic.

2. a) Calculate the mean molecular weights per particle, per electron and per ion for three chemical compositions: i) pure hydrogen ii) solar chemical composition, X = 0.7, Y=0.28, Z=0.02 and iii) for primeval chemical composition, X=0.76, Y=0.24, Z=0. Show that the three molecular weights always satisfy the relation $1/\mu_e + 1/\mu_i = 1/\mu$.
   b) Estimate the electron pressure, ion pressure and radiation pressure at the center of the Sun. Use a temperature $T_c = 1.6 \times 10^7$, density $\rho_c = 1.5 \times 10^5$ kg m^{-3} and composition (ii).
   c) Calculate the pressure of a non-relativistic, degenerate electron gas with density $\rho = 10^{10}$ kg m^{-3}. Derive an upper limit for the temperature of the gas.

3. a) A low mass red giant star has a degenerate helium core in which all the hydrogen has been burnt. The degenerate electron pressure, p_e, obeys the relation

$$p_e = 2.33 \times 10^{-38} n_e^{5/3} \ \mathsf{Nm}^{-2}. \tag{1}$$

   Helium starts to burn in the core at a temperature of 10^8 K. Show that, if the density in the core is greater than 8.8×10^7 kg m^{-3}. burning will commence in degenerate conditions, and that the core will undergo a helium flash. What temperature will be achieved in the flash before degeneracy is removed in a region which remains at a constant density of 5×10^8 kg m^{-3}?
   b) The star has an initial mass small enough that (after all the helium has been burnt in the flash) it cannot reach the temperatures needed for Carbon-burning and will turn into a white dwarf. Using the equation of hydrostatic equilibrium, derive an approximate expression for the pressure at the center of the star (use a constant density model). Derive a mass-radius relation for the white dwarf which is supported by electron degeneracy pressure ($P_e = k\rho^{5/3}$) What does the mass-radius relation for a white dwarf imply?