33-467: Astrophysics of Stars and the Galaxy
Due: Monday 12th September.
Problems 1 and 2 are worth 5 points. Problem 3 and 4, 10 points.

# Problem Set 1

1. Given that the luminosity of the Sun is $3.8 \times 10^{33}$ erg s$^{-1}$ and that the bolometric magnitude of the Sun is $M = 4.7$, estimate the distance at the which the Sun could just be seen by the naked eye (the naked eye can detect a star of apparent magnitude 6). Estimate the number of photons incident on the eye per second in this situation.

2. (a) Sketch the HR diagram for an old globular cluster, label the main sequence, the red giant and horizontal branch.

   (b) Using the observed mass-luminosity relation for nearby stars argue that stars must be forming at the present day.

   (c) The main sequence of the Pleiades cluster consists of stars with mass less than $6M_\odot$; the more massive stars have already evolved off the main sequence. Estimate the age of the Pleiades cluster.

3. Suppose that the density of a star varies linearly from the center to the surface,

$$\rho = \rho_c \left(1 - \frac{r}{R}\right), \tag{1}$$

   where $\rho_c$ is the central density and $R$ the stellar radius. Use the equation of mass continuity to show that

$$m(r) = \frac{4\pi}{3}\rho_c r^3 \left(1 - \frac{3r}{4R}\right). \tag{2}$$

   and find the total mass of the star. Hence show that $\rho_c = 4\bar{\rho}$, where $\bar{\rho}$ is the mean stellar density.

   Use the equation of hydrostatic equilibrium to show that the central pressure of the star is given by

$$P_c = \frac{5}{4\pi}\frac{GM^2}{R^4}, \tag{3}$$

   and hence obtain a numerical value of the Sun's central pressure and temperature-with this model (assuming that the equation of stat is that of a perfect gas lawwith $\mu = 0.61$, the mean molucular weight for "cosmic abundances").

4. The equation of motion of a mass element at the surface of a star of mass $M$, that has no internal pressure is

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2}. \tag{4}$$

   Show that as the star collapses from an initial radius R to some new radius r, the mass element acquires a velocity

$$v = -\sqrt{2GM(\frac{1}{r} - \frac{1}{R})}. \tag{5}$$

Hence show that the time required for the star to collapse to a point is

$$t = \frac{\pi}{2}\sqrt{\frac{R^3}{2GM}} \tag{6}$$

What is this time for a star of $3M_\odot$ which initially has a radius of $10^{-2}R_\odot$?